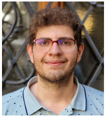
Partial Observation: Recent computational results





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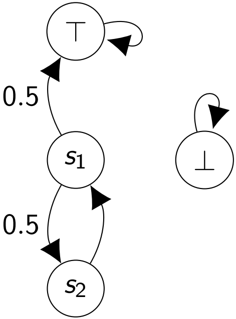
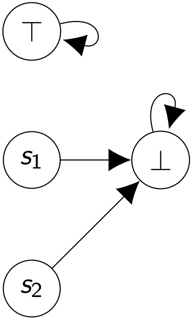
Probabilistic Finite Automata



Letter a



Letter b

Processing a letter defines the probabilistic transition.

Probabilistic Finite Automata: Language

The language of a Probabilistic Finite Automata is

$$\mathcal{L} := \left\{ w \in \Sigma^* : \mathbb{P}_{s_1}(S_{|w|} = \top) > 1/2 \right\} .$$

(In the previous example, $\mathcal{L} = aa\Sigma^*$)

The computational problem we consider is EMPTYNESS.

$$\mathcal{L} \stackrel{?}{=} \emptyset \, .$$

Probabilistic Finite Automata: Decidability

Theorem (Madani 2003)

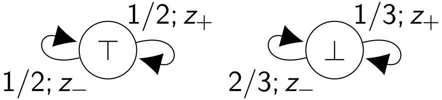
Deciding EMPTYNESS for Probabilistic Finite Automata is undecidable.

Theorem (Madani 2003)

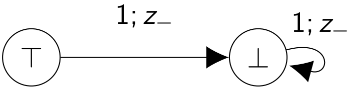
Deciding EMPTYNESS for Probabilistic Finite Automata where every word is accepted with probability in $[0, \varepsilon] \cup [1 - \varepsilon, 1]$ *is undecidable.*

Game Theoretical view

Partially Observable Markov Decision Process



Action win



Action lose

States s
Actions a
Signals z
Rewards $r(s, a)$
Stochastic transitions $\delta : S \times A \to \Delta(S \times Z)$

Partially Observable MDP: Objectives

- **Reachability**: eventually reach a state
- **Büchi/co-Büchi**: reach a state infinitely/finitely often
- **Limit average**: limit of the aggregated stage reward

$$\liminf_{m\to\infty} \frac{1}{m} \sum_{i=1}^{m} r(s_i, a_i) \qquad \limsup_{m\to\infty} \frac{1}{m} \sum_{i=1}^{m} r(s_i, a_i)$$

- **Limit reward**: limit of the stage reward

$$\liminf_{m\to\infty} r(s_m, a_m) \qquad \limsup_{m\to\infty} r(s_m, a_m)$$

Partially Observable MDP: Computational problems

- **Qualitative Reachability**
  Is the reachability value 1?
- **Synthesis of strategies**
  Compute an ε-optimal strategy
- **Value approximation**
  Approximate the value
- **Property checking**
  Is my POMDP particularly easy to solve?

Our results

Qualitative reachability

Theorem (2024)

Deciding qualitative reachability for POMDPs over constant memory (mixed) strategies is NP-complete.

Recall that

- over arbitrary strategies it is undecidable
- Existential Theory of the Reals upper bound is simple

Approximately optimal strategies

Theorem (2021)

Consider POMDPs with liminf average objective.
For all $\varepsilon > 0$, there exists a **finite-memory ε-optimal strategy**.

Theorem (2021)

Consider blind MDPs with liminf average objective.
For all $\varepsilon > 0$, there exists a **finite-recall ε-optimal strategy**.

Recall that

- value approximation is undecidable
- with this, value approximation is recursively enumerable

Approximation of the value

Theorem (2024)

Consider blind MDPs with liminf average objective.
If the transitions are ergodic, then
value approximation is EXP2SPACE. In particular, decidable.

Recall that

- value approximation is undecidable in general
- in the ergodic subclass, it is decidable

Continuity concepts

- Value-continuity
  Value of similar POMDPs is close

- Weak strategy-continuity
  Some approximately-optimal strategy is still approximately-optimal in similar POMDPs

- Strong strategy-continuity
  All approximately-optimal strategies are approximately-optimal in similar POMDPs

Continuity: checking

Theorem (to appear)

Deciding whether a POMDP is
value-, weakly strategy-, or strongly strategy-continuous
is **undecidable***.*

Continuity: separation of models

Model	Continuity		
	Value	Weak strategy	Strong strategy
Fully-observable MDPs	Yes	Yes	**No**
POMDPs	**No**	**No**	**No**
Blind MDPs	**Yes**	**Yes**	**Yes**

Thank you!